Vibration Suppression via Tunable Periodical Structural Design

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Annual Progress Report

Vibration Suppression via Tunable Periodical Structural Design

(Project #2005/5)

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Synopsis

It is known that a periodic structure acts as a band-pass filter to broad frequency band vibrations. Through the study of a tunable periodical structure with a simple periodical pattern under bending vibration, it is found that the filtering characteristics can be tuned as desired. In this study, a tunable periodical structure subject to longitudinal and torsional vibrations is examined. The filtering characteristics of a flexural vibrating periodical structure with complex periodical patterns are also investigated.

1. Background

Vibrations in mechanical structures are harmful. They not only greatly deteriorate a system's performance, but they also hasten the fatigue failure of a structure by repeated stresses. Vibrations can be controlled actively; however, industries such as the automotive industry are reluctant to add active controllers to their structures due to cost considerations.

A periodic structure is found to act as a band pass filter to broad frequency band vibrations. There exists a propagation zone and an attenuation zone in a periodic structure [1-5]. Most previous studies focused on simple flexural vibrating structures with fixed periodical parameters. In this project, a tunable periodical structure subject to longitudinal and torsional vibrations is examined. The filtering characteristics of flexural vibrating periodical one-dimensional structures with complex periodical patterns are also examined.

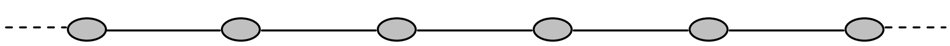
2. Objectives

In the past few decades, great efforts have been made by the automotive industry to isolate steering columns, drivelines and chassis systems from various excitation sources. However, vehicle vibration remains a substantial quality problem. The proposed project aims to attack the problem from a novel standpoint. Vibrations are suppressed through designing tunable periodical structures. The proposed approach is economical, since no active control components are involved.

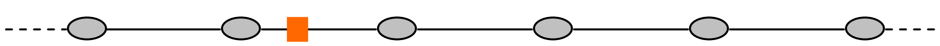
3. Approach

3.1 Introduction

Through studying periodical structures of simple patterns, such as that shown in Figure 1(a), it was found that periodical structures act as filters to incoming vibrations [1]. However, in practice, a periodical pattern can easily be broken by a structural modification or local repair. Figure 1(b) illustrates an example in which the original periodical pattern of Figure 1(a) is broken by a local structural modification, which makes the original periodical structure lose its filtering characteristics. This research explores the reconstruction of the "filtering" characteristics by rebuilding a more complex periodical pattern. Figure 1(c) shows a reconstructed periodical structure. There are multiple ways of reconstructing periodical patterns of a locally modified/repaired periodical structure. The preferred pattern depends on the desired filtering characteristics.



(a) Simple periodical pattern



(b) Periodical pattern broken by a local structural modification

(c) Reconstructed periodical pattern via pseudo-repair

Figure 1. Periodical structures.

Vibration analysis of a complex periodical structure makes it difficult for the conventional modal approach to be applied in solving the related vibration problems. An alternative to the modal vibration description is to describe vibrations as structural waves. From a wave standpoint, vibrations propagating along a waveguide are reflected and transmitted upon discontinuities and boundaries. The propagation is governed by the so-called propagation matrix, and the reflection and transmission by the reflection and transmission matrices [6-11]. Combining these matrices provides a concise and systematic approach to vibration analysis, regardless the complexity of the structure.

3.2 Wave Approach

3.21 Longitudinal and Torsional Vibration Analysis

Consider a beam or rod vibrating in the longitudinal direction. The equation of motion is:

$$EA\frac{\partial^2 u}{\partial x^2} - \rho A\frac{\partial^2 u}{\partial t^2} = 0 \tag{1}$$

where $u(x,t)$ is the longitudinal displacement, E Young's modulus, ρ the density, and A the cross sectional area of the beam.

Consider a beam or rod vibrating in torsion. The equation of motion is:

$$GJ\frac{\partial^2 \theta}{\partial x^2} - \rho J\frac{\partial^2 \theta}{\partial t^2} = 0 \tag{2}$$

where $\theta(x,t)$ is the rotational angle, G the shear modulus, and J the polar moment of inertia of the cross sectional area of the beam.

It is not difficult to see that the above two equations of motion are of the same form. As a result, the analysis below focuses on longitudinal vibration.

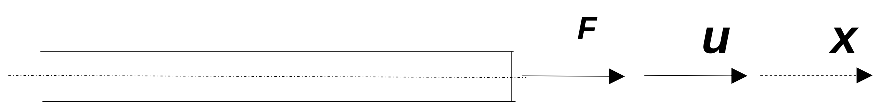


Figure 2. Definition of positive longitudinal force.

The corresponding longitudinal force F as defined in Figure 2 is given by:

$$F = -EA\frac{\partial u}{\partial x} \tag{3}$$

Considering time harmonic motion, the solutions to Eq. (1) can be expressed as follows:

$$u = c^+ e^{-ik_1 x} + c^- e^{ik_1 x} \tag{4}$$

where the time dependence $e^{i\omega t}$ has been suppressed. $k_1 = \sqrt{\rho\omega^2 / E}$ is the longitudinal wave number and ω is the circular frequency. Eq. (4) indicates that there exists a positively and a negatively going longitudinal wave in the structure.

Waves propagate from one point to another along a uniform structure (waveguide). Denoting the positively and negatively going waves at points A and B as $\mathbf{a}^+$, $\mathbf{a}^-$, $\mathbf{b}^+$, and $\mathbf{b}^-$, respectively, they are related by

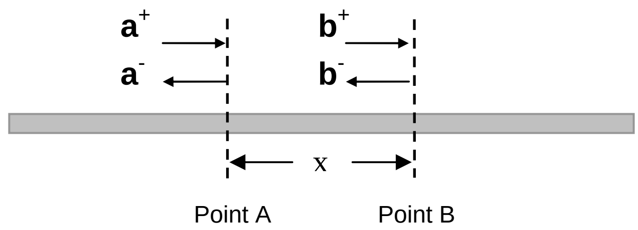


Figure 3. Wave propagation.

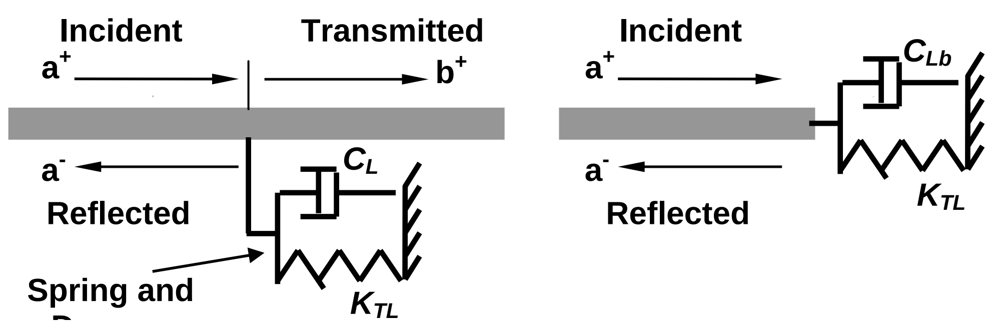
$$\mathbf{a}^- = \mathbf{f}(x)\mathbf{b}^-; \quad \mathbf{b}^+ = \mathbf{f}(x)\mathbf{a}^+ \tag{5}$$

where $\mathbf{f}(x)$ is the propagation matrix for a distance x. $\mathbf{f}(x) = e^{-ik_1 x}$ for longitudinal vibration.

The reflection and transmission characteristics at a discontinuity are governed by the reflection and transmission matrices, which are specific discontinuity-dependent. Likewise, the reflection characteristics at a boundary are governed by the reflection matrix, which is the specific boundary-dependent. The reflection and transmission matrices can be derived based on local equilibrium and continuity conditions.

The reflected and transmitted waves on both sides of the discontinuity as shown in Figure 4(a) are related to the incident vibration wave as follows:

$$\mathbf{a}^- = \mathbf{r}\mathbf{a}^+, \; \mathbf{b}^+ = \mathbf{t}\mathbf{a}^+ \tag{6}$$



(a) A discontinuity (b) A boundary

Figure 4. Wave reflection and transmission for longitudinal vibration.

The continuity of the beam at a discontinuity requires that

$$u_- = u_+ \tag{7}$$

Therefore,

$$\mathbf{a}^+ + \mathbf{a}^- = \mathbf{b}^+ \tag{8}$$

The equilibrium of the support requires

$$K_{TL}u_{\pm} + C_L \dot{u}_{\pm} = F_+ - F_- \tag{9}$$

where K_{TL} and C_L are the stiffness and damping constant of the longitudinal tunable support. Thus,

$$i\mathbf{a}^+ - i\mathbf{a}^- - (K_{TL} + i\omega C_L + i)\mathbf{b}^+ = 0 \tag{10}$$

Equations (6), (8), and (10) can be solved to obtain the reflection and transmission matrices, which are given as

$$\mathbf{r}_L = -\frac{K_{TL} + i\omega C_L}{2i + K_{TL} + i\omega C_L} \tag{11a}$$

$$\mathbf{t}_L = \frac{2i}{2i + K_{TL} + i\omega C_L} \tag{11b}$$

Similarly, at the boundary as shown in Figure 4(b), there exists the following relationship:

$$\mathbf{a}^- = \mathbf{r}_b \mathbf{a}^+ \tag{12}$$

The equilibrium of the boundary support requires that

$$F = K_{TLb}u + C_{Lb}\dot{u} \tag{13}$$

From the above equations, the reflection coefficient at the boundary is found as

$$\mathbf{r}_{Lb} = \frac{i - K_{TLb} - i\omega C_{Lb}}{i + K_{TLb} + i\omega C_{Lb}} \tag{14}$$

3.21 Transverse Vibration Analysis

Transverse vibrations of a simple tunable periodical beam structure have been studied using the wave approach [11]. For the reader's convenience, the equation of motion and the propagation, reflection, and transmission matrices are listed below.

Consider a thin beam lying along the *x*-axis. The equation of motion according to Euler-Bernoulli beam theory is

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \tag{15}$$

where EI is the flexural rigidity of the beam and w is the flexural deflection.

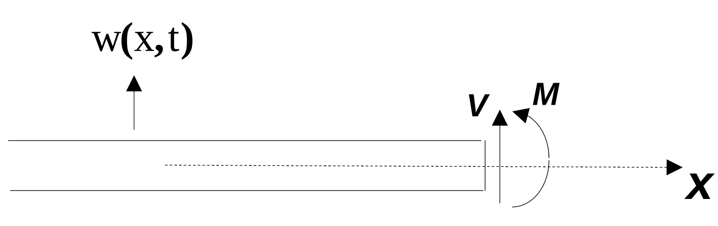


Figure 5. Definition of positive shear force and bending moment.

The corresponding shear force V and bending moment M as defined in Figure 5 are

$$V = -EI\frac{\partial^3 w}{\partial x^3},\ M = -EI\frac{\partial^2 w}{\partial x^2} \tag{16}$$

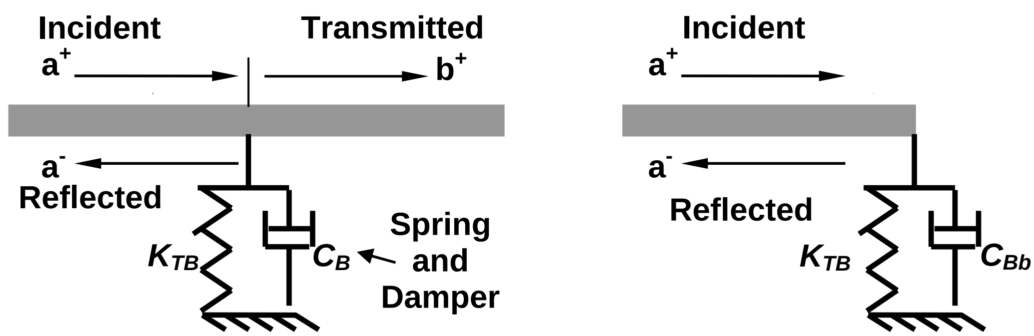
Considering time harmonic motion, with the time dependence $e^{i\omega t}$ suppressed, the solution to equation (15) can be expressed in wave form as

$$w = a^+ e^{-ik_2 x} + a_N^+ e^{-k_2 x} + a^- e^{ik_2 x} + a_N^- e^{k_2 x} \tag{17}$$

where $k_2 = \sqrt[4]{\rho A\omega^2/(EI)}$ is the transverse wave number.

The propagation matrix is

$$\mathbf{f}(x) = \begin{bmatrix} e^{-ik_2 x} & 0 \\ 0 & e^{-k_2 x} \end{bmatrix} \tag{18}$$



(a) A discontinuity (b) A boundary

Figure 6. Wave reflection and transmission for transverse vibration.

The reflection and transmission matrices at a discontinuity with transverse support stiffness K_{TB} and damping C_B as shown in Figure 6(a) are

$$\mathbf{r}_B = \begin{bmatrix} \dfrac{0.5(1+i)(iK_{TB}-\omega C_B)}{2+\omega C_B + i(2-K_{TB})} & \dfrac{0.5(1+i)(iK_{TB}-\omega C_B)}{2+\omega C_B + i(2-K_{TB})} \\ \\ \dfrac{0.5(1-i)(iK_{TB}-\omega C_B)}{2+\omega C_B + i(2-K_{TB})} & \dfrac{0.5(1-i)(iK_{TB}-\omega C_B)}{2+\omega C_B + i(2-K_{TB})} \end{bmatrix} \tag{19a}$$

$$\mathbf{t}_B = \begin{bmatrix} \dfrac{0.5(1-i)(\omega C_B + i(4-K_{TB}))}{2+\omega C_B + i(2-K_{TB})} & \dfrac{0.5(1+i)(iK_{TB}-\omega C_B)}{2+\omega C_B + i(2-K_{TB})} \\ \\ \dfrac{0.5(1-i)(iK_{TB}-\omega C_B)}{2+\omega C_B + i(2-K_{TB})} & \dfrac{0.5(1+i)(\omega C_B + 4 - iK_{TB})}{2+\omega C_B + i(2-K_{TB})} \end{bmatrix} \tag{19b}$$

The reflection matrix at a boundary as shown in Figure 6(b) is

$$\mathbf{r}_{Bb} = -\begin{bmatrix} K_{TBb} + i\omega C_{Bb} - i & K_{TBb} + i\omega C_{Bb} + 1 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} K_{TBb} + i\omega C_{Bb} + i & K_{TBb} + i\omega C_{Bb} - 1 \\ -1 & 1 \end{bmatrix} \quad (20)$$

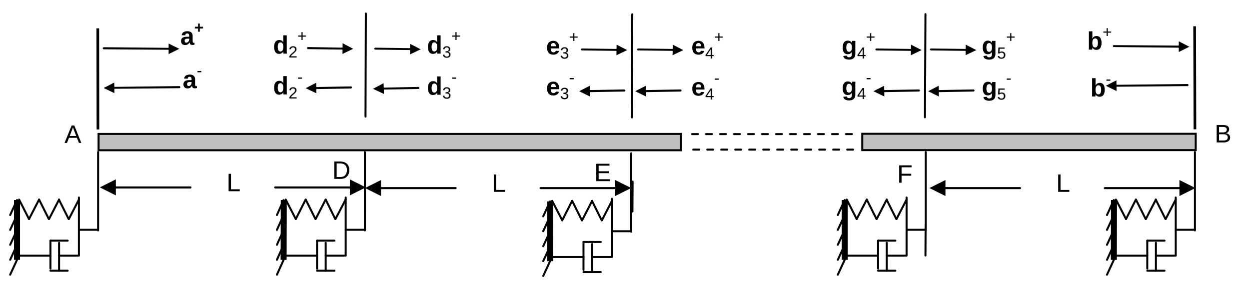
3.3 Vibration Analysis of Periodical Structures using Wave Approach

In this study, longitudinal and bending vibrations of both simple (Figure 7(a)) and complex periodical structures (Figure 7(b)) are investigated.

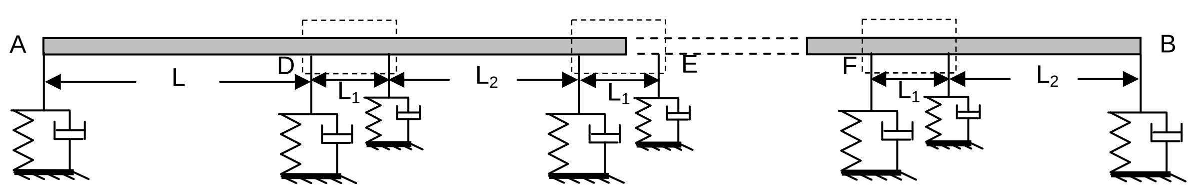
3.31 Longitudinal Vibration Analysis of a Simple Periodical Structure

For a simple periodical structure with N discontinuities, there are (N+1) pairs of propagation relations, N sets of reflection and transmission relations at the discontinuities, and two boundary reflection relations involved in the analysis. These equations are assembled in matrix form as shown in Eq. (21).

By setting the determinant of the coefficient matrix to zero, one obtains the characteristic equation (CE) from which the natural frequencies of the periodical distributed structure are found.



(a) Simple periodical structure with longitudinal supports.



(b) Complex periodical structure with transverse supports.

Figure 7. Periodical structures.

$$\begin{bmatrix}
-1 & \mathbf{r}_a & 0 & 0 & & & & & & \\
0 & -1 & 0 & \mathbf{f}(L) & & & & & & \\
\mathbf{f}(L) & 0 & -1 & 0 & & & & & & \\
 & & \mathbf{Q} & & & & & \mathbf{0} & & \\
 & & & \mathbf{Q} & & & & & & \\
 & & & & \ddots & & & & & \\
 & & & & & \ddots & & & & \\
 & & & & & & \mathbf{Q} & & & \\
 & \mathbf{0} & & & & & & \mathbf{Q} & & \\
 & & & & & & & & \mathbf{Q} & \\
 & & & & & & & & \mathbf{r}_b & -1
\end{bmatrix}
\begin{bmatrix}
\mathbf{a}^+ \\ \mathbf{a}^- \\ \mathbf{d}_2^+ \\ \mathbf{d}_2^- \\ \mathbf{d}_3^+ \\ \mathbf{d}_3^- \\ \vdots \\ \vdots \\ \mathbf{g}_4^+ \\ \mathbf{g}_4^- \\ \mathbf{g}_5^+ \\ \mathbf{g}_5^- \\ \mathbf{b}^- \\ \mathbf{b}^+
\end{bmatrix} = 0 \tag{21}$$

where

$$\mathbf{Q} = \begin{bmatrix}
\mathbf{r} & -1 & 0 & \mathbf{t} & 0 & 0 \\
\mathbf{t} & 0 & -1 & \mathbf{r} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & \mathbf{f}(L) \\
0 & 0 & \mathbf{f}(L) & 0 & -1 & 0
\end{bmatrix} \tag{21a}$$

3.32 Vibration of a Complex Periodical Structure

A periodical structure with complex periodical pattern can be analyzed in a similar way as described above by simply grouping the complex discontinuities. The reflection and transmission of the grouped discontinuity is found by considering the reflection and transmission characteristics of the discontinuities involved in the group, as well as the propagation relations between/among the discontinuities within the group.

In general, the grouped discontinuity is asymmetric with regard to a wave incident from the left and right hand sides of the grouped discontinuity. Two sets of reflection and transmission matrices need to be derived around a grouped discontinuity. Considering right hand side incidence as shown in Figure 8, one has:

$$\mathbf{b}^+ = \mathbf{t}_{gr}\mathbf{a}^+ ; \quad \mathbf{a}^- = \mathbf{r}_{gr}\mathbf{a}^+ \tag{22}$$

where $\mathbf{t}_{gr}$ and $\mathbf{r}_{gr}$ are the transmission and reflection matrices of the discontinuity group, and are found as

$$\mathbf{t}_{gr} = \left[\mathbf{t}_1^{-1} * (\mathbf{f}^{-1}(L_1) * \mathbf{t}_2^{-1} - \mathbf{r}_1 * \mathbf{f}^{-1}(L_1) * \mathbf{r}_2 * \mathbf{t}_2^{-1})\right]^{-1} \tag{23a}$$

$$\mathbf{r}_{gr} = \mathbf{r}_1 + \mathbf{t}_1 * \mathbf{f}^{-1}(L_1) * \mathbf{r}_2 * \mathbf{t}_2^{-1} * \mathbf{t}_{gr} \tag{23b}$$

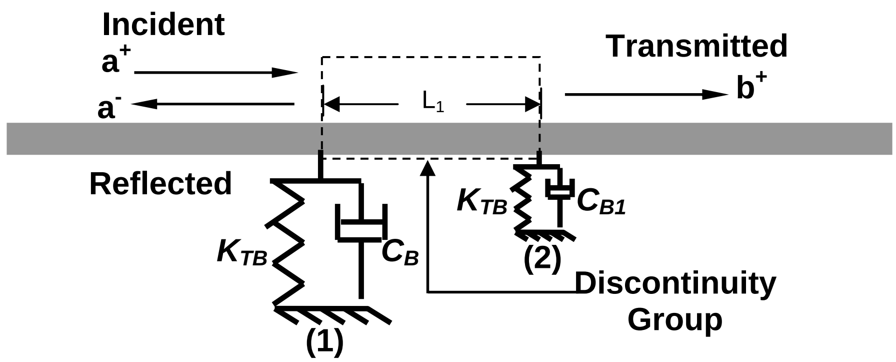


Figure 8. Discontinuity group.

Similarly, the reflection and transmission matrices of the discontinuity group with left hand side incidence are:

$$\mathbf{t}_{gl} = \left[\mathbf{t}_2^{-1} * (\mathbf{f}^{-1}(L_1) * \mathbf{t}_1^{-1} - \mathbf{r}_2 * \mathbf{f}^{-1}(L_1) * \mathbf{r}_1 * \mathbf{t}_1^{-1})\right]^{-1} \tag{24a}$$

$$\mathbf{r}_{gl} = \mathbf{r}_2 + \mathbf{t}_2 * \mathbf{f}^{-1}(L_1) * \mathbf{r}_1 * \mathbf{t}_1^{-1} * \mathbf{t}_{lr} \tag{24b}$$

The analysis is similar to that of a simple periodical pattern. The only change is in the expression of Q in Eq. (21), which is now

$$\mathbf{Q} = \begin{bmatrix} \mathbf{r}_{gr} & -\mathbf{1} & \mathbf{0} & \mathbf{t}_{gl} & \mathbf{0} & \mathbf{0} \\ \mathbf{t}_{gr} & \mathbf{0} & -\mathbf{1} & \mathbf{r}_{gl} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1} & \mathbf{0} & \mathbf{f}(L_2) \\ \mathbf{0} & \mathbf{0} & \mathbf{f}(L_2) & \mathbf{0} & -\mathbf{1} & \mathbf{0} \end{bmatrix} \tag{25}$$

4. Numerical Results

With the availability of propagation, reflection, and transmission matrices as derived above, vibration analysis of complex periodical structures become concise and systematic. Table 1 lists the physical parameters of an example structure. For longitudinal vibration analysis, unless otherwise stated, the boundary conditions of the beam are clamped-clamped.

Table 1. Physical parameters of an example structure.

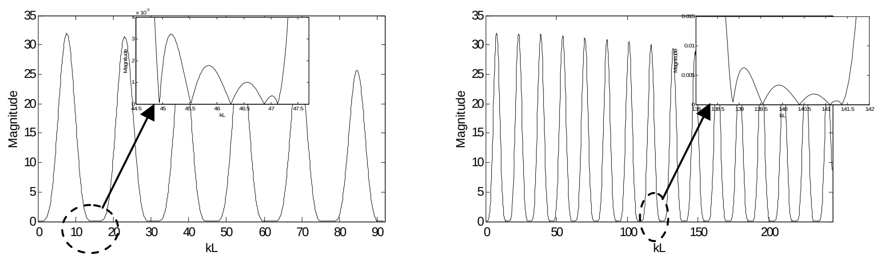
Density (kg/m^3)	Young's Modulus (GPa)	Width (mm)	Thickness (mm)
7800	206	10	10

4.1 Longitudinal Vibration Analysis of a Simple Periodical Structure

Effects of the following parameters on the dynamic characteristics of simple periodical structures are studied from a wave standpoint: (1) length of bay; (2) number of bays; (3) boundaries; and (4) intermediate supports.

(1) Length of bay (N+1)

It has been found that, the longer the bay, the more passing/stopping bands exist within a given frequency range, which in this case is 0-10000Hz. When kL rather than frequency f is chosen as the variable, the bands are of equal width, as shown in Figure 9.

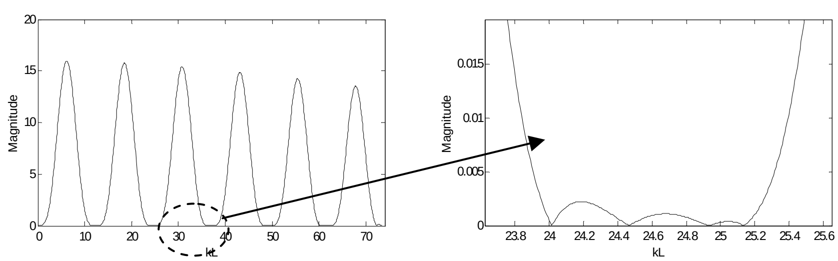


(a) L=1.5 m, N=4 (b) L=4 m, N=4

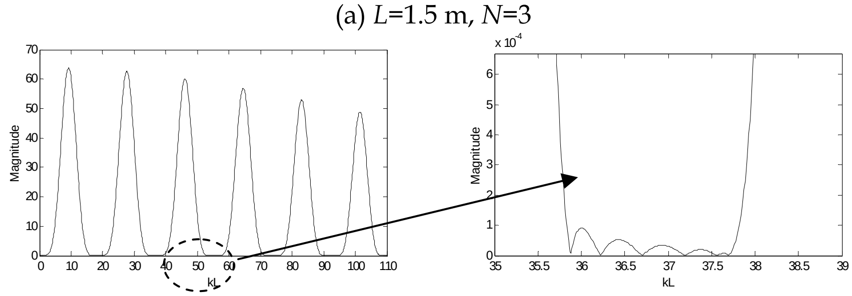
Figure 9. Effect of length of bay on natural frequencies.

(2) Number of bays

The number of bays hardly affects the passing/stopping band distribution. However, the number of natural frequencies within a passing band varies. It equals the number of bays, as shown below in Figure 10.



(a) L=1.5 m, N=3



(b) L=1.5 m, N=5

Figure 10. Effect of number of bays (N+1) on natural frequencies.

(3) Boundary

Boundaries affect the distribution of the passing/stopping bands. As an example, Figure 11 shows the responses of an L=1.5 m and N=4 periodical beam with varying boundary conditions. An interesting observed phenomenon is that a free boundary moves one of the natural frequencies away from a stopping band and places that natural frequency in between two stopping bands. This narrows the width of stopping band as a result.

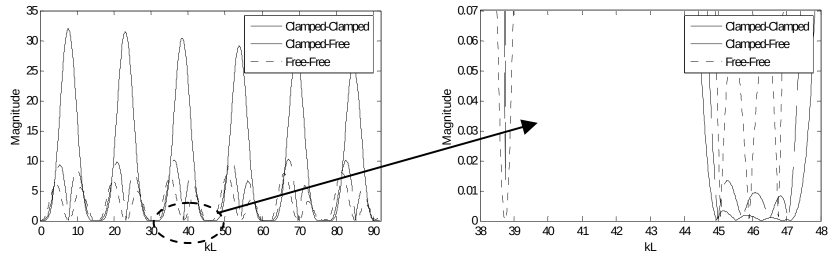
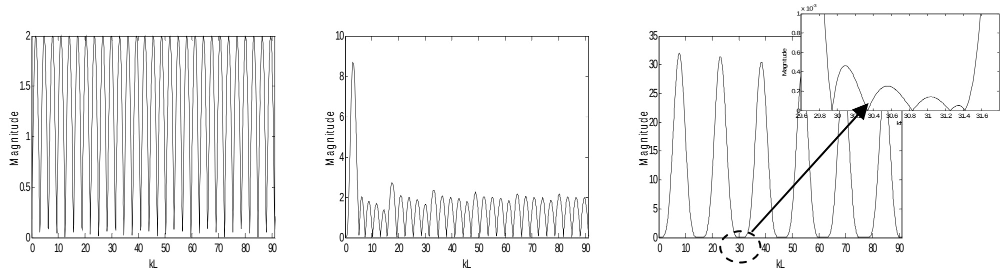
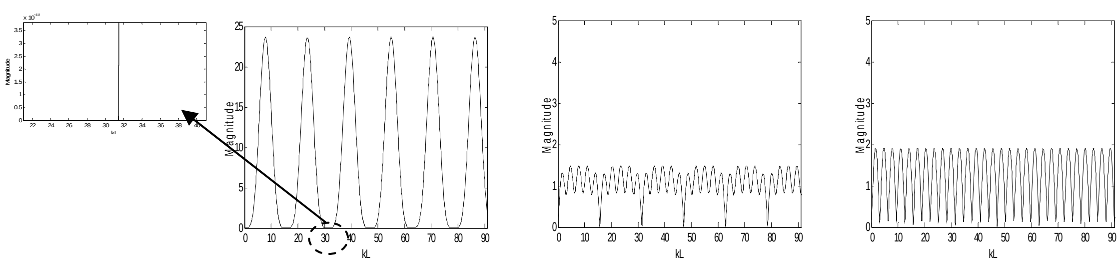


Figure 11. Effect of boundaries on natural frequencies.



(a) K_{TL} =0, C_L =0 (b) K_{TL} =10^7, C_L =0 (c) K_{TL} =10^9, C_L =0

Figure 12. Effect of support stiffness on natural frequencies.



(a) K_{TL} =0, C_L =10^5 (b) K_{TL} =0, C_L =10^3 (c) K_{TL} =0, C_L =0

Figure 13. Effect of support damping on natural frequencies.

(4) Intermediate support

The intermediate supports considered here are general spring-damper supports, with the spring stiffness and damping constant being tunable. In the first case, the damping constant is assumed to be zero, with spring stiffness varying from zero to infinity. The responses are shown in Figure 12. As the stiffness increases, a dynamic forming process of passing/stopping bands is observed. The band is seen to initiate from lower frequencies and get pushed further to higher frequencies as the stiffness increases.

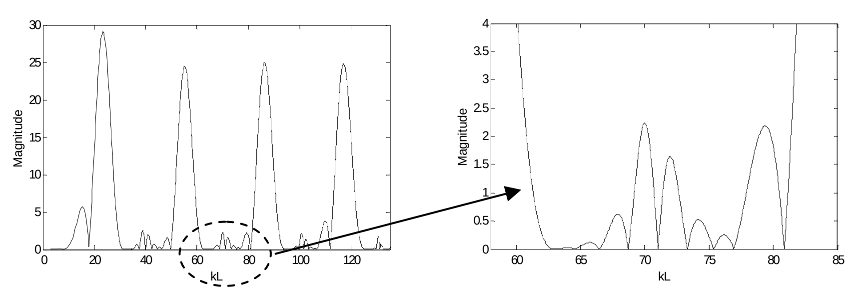
In the second case, the spring stiffness is assumed to be zero, with the damping constant varying. It was found that there exists a critical damping value. Below the critical value, there exists no stopping/passing band. But above this critical frequency, wide stopping bands are formed, and each passing band consists of a single passing point. This state may be favorable for vibration suppression purposes. Figure 13 demonstrates the dynamic process. The example beam has L=1.5 m and N=5.

4.2 Transverse Vibration Analysis

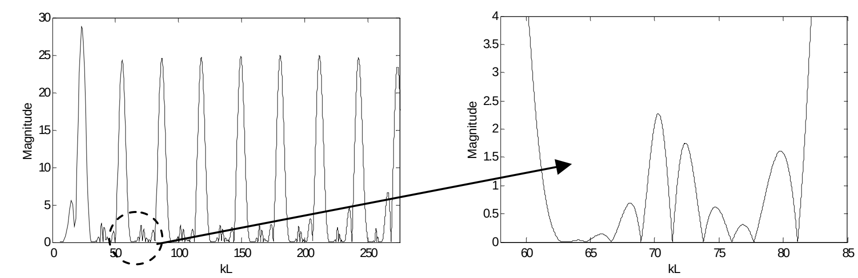
The reflection, transmission, and propagation matrices corresponding to bending vibrations are listed in Section 3.2.2. More details can be found in [11]. In the examples considered in this section, the boundary conditions are assumed to be simply supported-simply supported. Here (N+1) refers to the number of bays between the two discontinuity groups, including the boundaries.

(1) Length effect

Let us examine the complex periodical structure shown in Figure 7(b) in bending. Two length-related parameters are needed to describe the effect of length on natural frequencies, since $L=L_1+L_2$. Figure 14 shows how length L affects the filtering characteristics of the periodical structure. It was found that the longer the bay, the more passing/stopping bands exist within a given frequency band (the frequency range is from 0 to 10,000 Hz in this case), similar to the situation of a simple periodical structure [11].



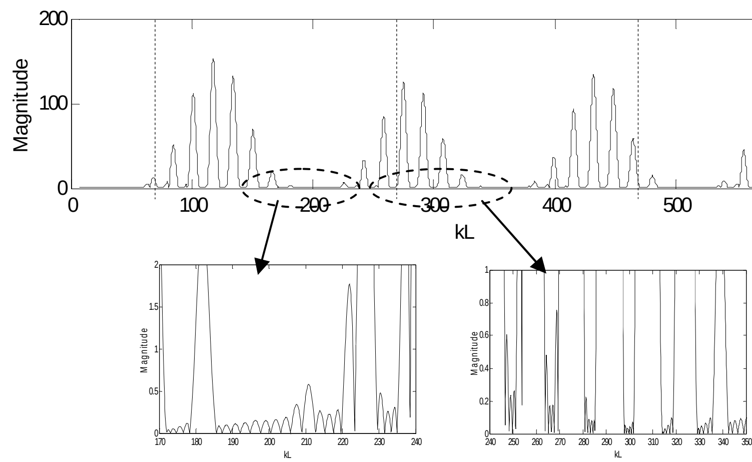
(a) L=0.3 m, N=4, L_1/L=0.5



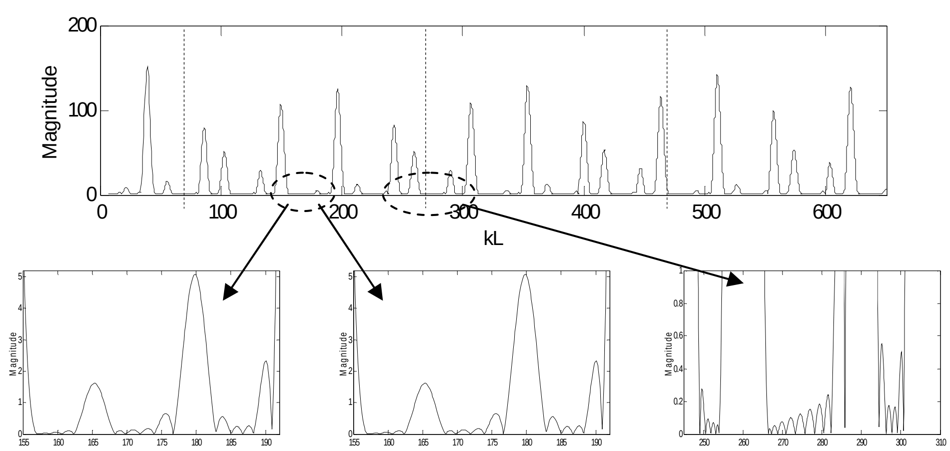
(b) L=0.6 m, N=4, L_1/L=0.5

Figure 14. Effect of bay length on natural frequencies.

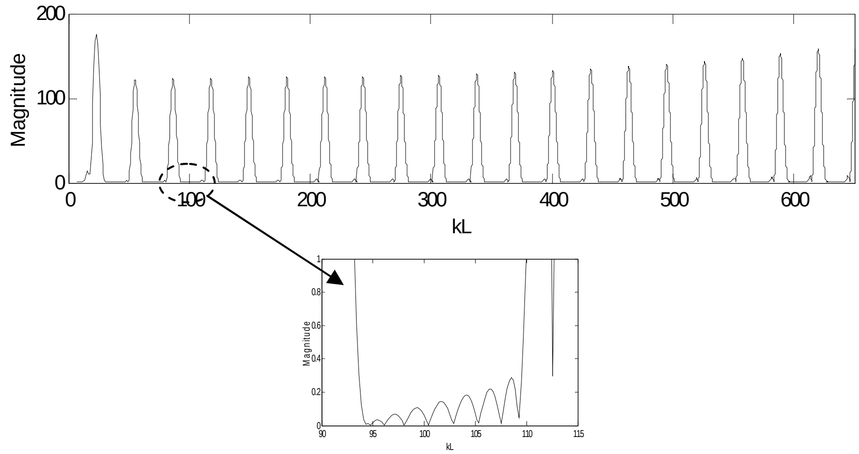
The effect of length ratio L_1/L is shown in Figure 15. It was found that to have a clear and wide stop band, a too small spacing between the two grouped discontinuities should be avoided.



(a) L_1/L =0.1, L=2 m, N=4



(b) L_1/L =0.3, L=2 m, N=4

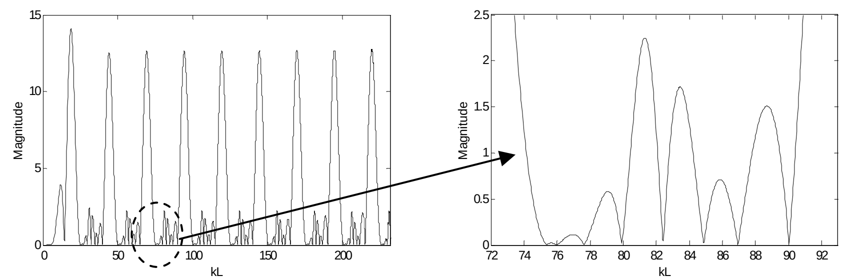


(c) L_1/L =0.5, L=2 m, N=4

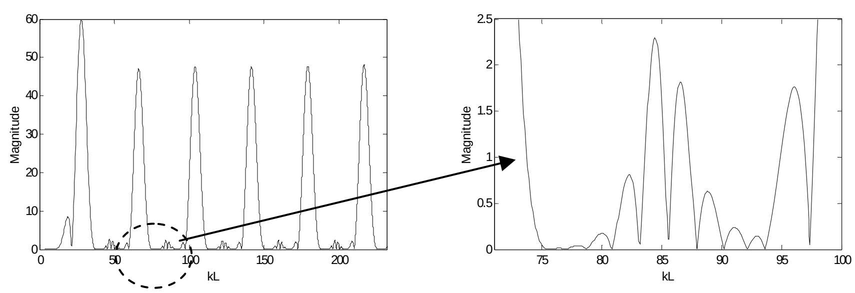
Figure 15. Effect of length ratio on natural frequencies.

(2) Number of bays (N+1)

The number of bays hardly affects the passing/stopping band distribution. However, it determines the number of natural frequencies within a passing band (Figure 16).



(a) N=3, L_1/L =0.5, L=0.9 m



(b) N=5, L_1/L =0.5, L=0.9 m

Figure 16. Effect of number of bays (N+1) on natural frequencies.

(3) Boundary

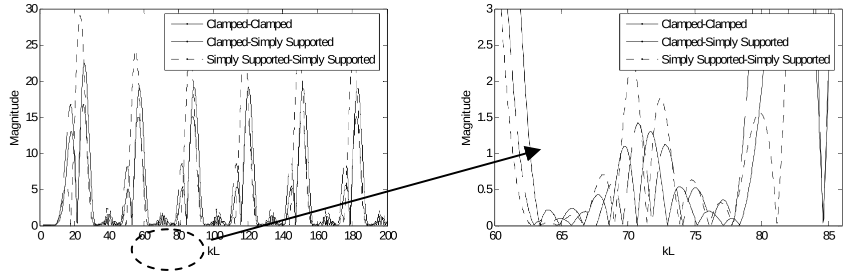


Figure 17. Effect of boundary conditions on natural frequencies, N=4, L_1/L =0.5, L=0.9.

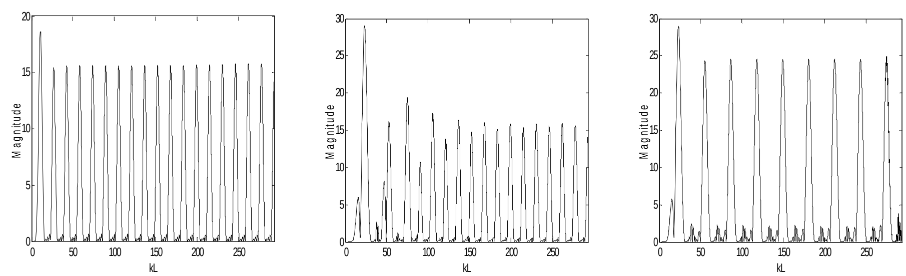
Figure 17 shows the responses of a complex periodical structure with varying boundary conditions. Boundaries are seen to slightly shift the passing/stopping band.

(4) Intermediate support

The intermediate supports considered here are general spring-damper supports, with spring stiffness and damping constant being tunable. The effect of stiffness and damping constants are studied. Other parameters are set as follows: N=4, L_1/L =0.5, L=0.9 m.

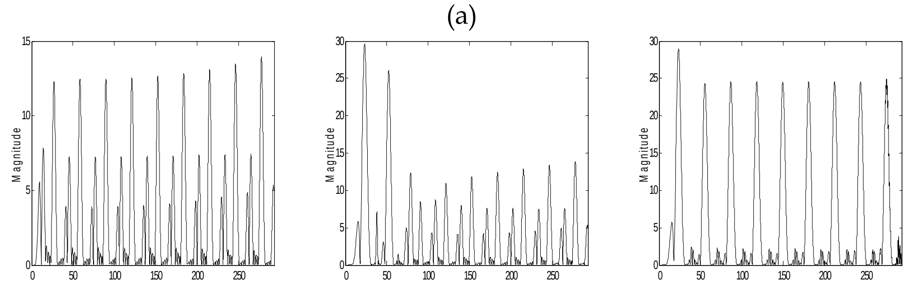
(i) Stiffness (K_{TB} , K_{TB1})

The damping constants are assumed to be zero. The stiffness of one spring is set to a fixed value, while the other is allowed to vary from zero to infinity. A dynamic process of changing passing/stopping bands is observed for both cases, as shown in Figure 18.



K_{TB1} =0, K_{TB} =10^{10} K_{TB1} =10^6, K_{TB} =10^{10} K_{TB1} =10^9, K_{TB} =10^{10}

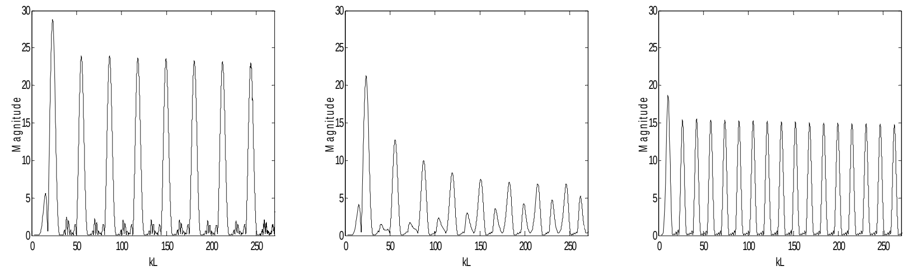
(a)



K_{TB1} =10^9, K_{TB} =0 K_{TB1} =10^9, K_{TB} =10^6 K_{TB1} =10^9, K_{TB} =10^{10}

(b)

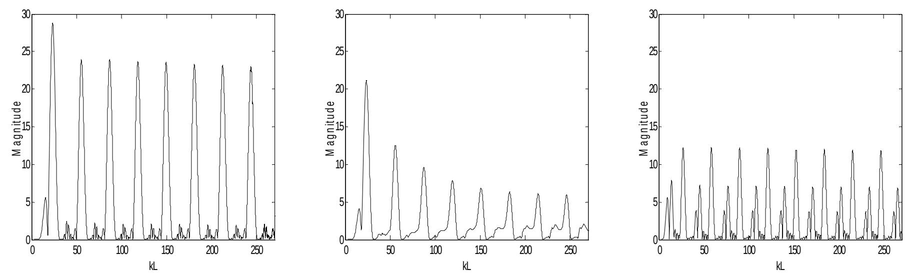
Figure 18. The effect of support stiffness on natural frequencies.



$C_{B1}=10^5$, $C_B=10^5$ $C_{B1}=10^3$, $C_B=10^5$ $C_{B1}=0$,

$C_B=10^5$

(a)



$C_{B1}=10^5$, $C_B=10^5$ $C_{B1}=10^5$, $C_B=10^3$ $C_{B1}=10^5$, $C_B=0$

(b)

Figure 19. The effect of support damping on natural frequencies.

(ii) Damping constants (C_B, C_{B1})

In this case, the stiffness is assumed to be zero. One of the damping constants is set to a fixed value, while the other is allowed to vary from zero to infinity. A dynamic process of changing passing/stopping bands is observed for both cases, as shown in Figure 19.

5. Conclusions

Vibration filtering characteristics of complex finite periodical structures have been studied. It was found that the length of the bay, number of bays, boundary conditions, stiffness ratio, damping constant ratio, spacing between discontinuities, etc., all affect the dynamic characteristics of a finite complex periodical structure. Hence, these parameters can be carefully designed as to allow the troublesome frequencies to fall into the stopping band; that is, to be filtered out. The spring damper intermediate supports allow the dynamic characteristics of an existing structure to be tuned for vibration suppression purposes.

6. Impact

6.1 Educational Impact

This research involves mechanical measurements and structural design. It aims to control vibrations. Hence, research results serve as very good practical problem solving examples for ME 540

Mechanical Vibrations, ME 442 Control Systems Analysis and Design, and ME 563 Advanced Instrument and Control.

6.2 Industrial Impact

The developed technique can be used in suppressing vibrations in vehicular systems. For example, a periodical structural design can be introduced in chassis systems, drivelines, and steering columns. Through the integration of structural design and control engineering, currently a very uncommon practice, this research offers an economical and practical solution to industrial noise vibration harshness-related problems.

7. Acknowledgment

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